LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

SECOND SEMESTER – APRIL 2010

ST 2812 - TESTING STATISTICAL HYPOTHESES

Date & Time: 19/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

SECTION – A

Answer ALL the following questions (10 x 2 = 20 marks)

1. Give an example for a multiple decision problem.
2. Define a Bayes decision rule.
3. Give an example of an invariant decision problem.
4. What is a randomized test?
5. Define power function of a test.
6. Define an unbiased test.
7. Define similar test.
8. State the correspondence between Acceptance Regions and Confidence Sets.
9. Give a testing situation where both 'Unbiasedness' and "Invariance' principles lead to the same optimum procedures.
10. State the test statistic for testing hypothesis about the variance of a normal population under the Likelihood Ratio criterion.

SECTION – B

Answer any FIVE of the following questions (5x 8 = 40 marks)

11. For the two decision problem with d_o: $\theta \in \Theta_o$ and d_1: $\theta \in \Theta_1$ and with loss function $L(\theta, d_o) = a$ for $\theta \in \Theta_1$, 0 for $\theta \in \Theta_o$ and $L(\theta, d_1) = b$ for $\theta \in \Theta_o$, 0 for $\theta \in \Theta_1$, show that every minimax procedure is unbiased.

12. Let $X = (X_1, \ldots, X_n)$ and suppose one wishes to test H: $X \sim f_1(x_1 - \theta, \ldots, x_n - \theta)$ versus K: $X \sim f_1(x_1 - \theta, \ldots, x_n - \theta)$. Obtain UMPIT under location transformations.

13. Using a random sample from U(0, θ) derive UMPT for H: $\theta \geq \theta_0$ versus K: $\theta < \theta_0$.

14. Let **X** have distribution $P \in \mathscr{P}$ and T be a sufficient statistic for $\mathscr{P}$. Show that a necessary and sufficient condition for all similar tests to have Neyman structure is that the family $\mathscr{P}^T$ of distributions of T is boundedly complete.

15. Obtain the UMPUT for H: $\sigma = \sigma_o$ versus K: $\sigma \neq \sigma_o$ in the case of normal distribution with known mean and deduce the 'side conditions' that are required to be satisfied.

16. If $X \sim B(m, p_1)$ and $Y \sim B(n, p_2)$ and are independent, derive UMPUT for H: $p_1 \leq p_2$ versus K: $p_1 > p_2$.

17. If λ denotes the likelihood ratio for testing a simple hypothesis H based on a random sample from a distribution, derive the asymptotic distribution of λ under H by stating the required assumptions.

18. Obtain the Likelihood Ratio Test for equality of means of 'k' normal populations with a common variance.

SECTION – C

Answer any TWO of the following questions (2x 20 = 40 marks)

19. (a) Let $X \sim P_\theta$ with MLR in T(x). Obtain the form of UMPT for H: $\theta \leq \theta_0$ versus K: $\theta > \theta_0$. Show that its power function $\beta_\varphi(\theta)$ is strictly increasing for all θ for which $0 < \beta_\varphi(\theta) < 1$. Apply this to obtain UMPT for one-parameter exponential family.
(b) Derive UMP test for one-sided hypotheses concerning the parameter of a Poisson Process under 'Inverse Poisson Sampling'. (14+6)

20. Let $f_1, f_2, \ldots, f_{m+1}$ be real-valued μ-integrable functions and let $\alpha_1, \alpha_2, \ldots, \alpha_m$ be given constants. Let $\mathscr{C} = \{ \phi \mid \int \phi\, f_i\, d\mu = \alpha_i,\ i = 1, 2, \ldots, m \}$ be a class of critical functions. Establish the existence of a critical function in $\mathscr{C}$, sufficient condition and necessary condition on the form of the critical function in $\mathscr{C}$ for maximization of $\int \phi f_{m+1}\, d\mu$.

21. (a) Discuss UMPUT for H: $\theta = \theta_0$ versus K: $\theta \neq \theta_0$ where θ is a single interesting parameter in a multiparameter exponential family.
(b) Obtain UMPUT for testing the independence of attributes in a 2 x 2 contingency table. (8+12)

22. Based on independent samples $X_1, \ldots, X_m$ from $N(\mu_1, \sigma_1^2)$ and $Y_1, \ldots, Y_n$ from $N(\mu_2, \sigma_2^2)$, consider the problem of testing H: $\sigma_2^2/\sigma_1^2 \leq \Delta_0$ versus K: $\sigma_2^2/\sigma_1^2 > \Delta_0$. Obtain (a) UMPUT and (b) UMPIT under a suitable group of transformations; and show that they coincide. (10+10)

* * * * * * *